Localization of Defects
and
Applications to Parameter Identification

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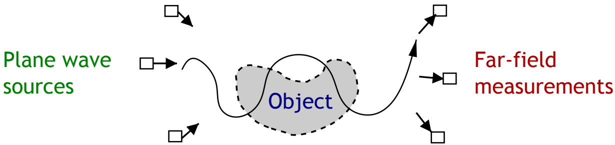


Figure: Acoustic scattering: plane wave incidence directions and far-field measurements.

- Problem : recover information about a scatterer from far field data

Goals

1. Reconstruct the scatterer's refraction index through an iterative numerical method
2. Build a fast numerical method to locate defects in some reference refraction index.
3. Investigate the coupling of these methods.

Mathematical setting

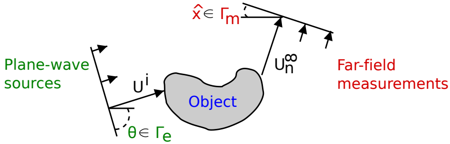


Figure: Inhomogeneous medium (O) studied at a fixed frequency

- Plane wave sources : $u^i(x) := e^{ikx\cdot\vec{\theta}}$, $x \in \mathbb{R}^d$, $\vec{\theta} \in \Gamma_e$
- Helmholtz equation for inhomogeneous media in an unbounded domain:

$$\begin{cases} \Delta u^s + k^2 n(x) u^s = -k^2(n(x) - 1)u^i, \; x \in \mathbb{R}^d, \\ \lim\limits_{|x|\to\infty} |x|^{\frac{d-1}{2}} (\partial_{|x|} u^s - iku^s) = 0. \end{cases}$$

- Far-field pattern : $u^s(x) = \frac{e^{ik|x|}}{|x|^{\frac{d-1}{2}}} u^\infty(\hat{x}) + o\left(\frac{1}{|x|^{\frac{d-1}{2}}}\right)$, $x \in \mathbb{R}^d$, $\hat{x} \in \Gamma_m$

- Problem: extract some information about the *actual* medium's index $n^\star \in L^\infty(O)$ from far-field measurements $u^\infty \in \mathcal{C}^\infty(\Gamma_e, \Gamma_m)$ and a *reference* medium's index $n \in L^\infty(O)$.
- Difficulties: non-linear and ill-posed inverse problem

Theorem

- $n(x), n^\star(x) \in \mathbb{R}$
- $(n - n^\star)(x) > 0$ or < 0
- Incoming and measurement directions covering the whole unit sphere

$$n(x) \neq n^\star(x) \iff 0 < \mathcal{M}_{\{n,n^\star\}}(x) := \| W^{-\frac{1}{2}} \overline{u(\cdot, x)} \|^{-2}_{L^2(\Gamma_e)}$$

where W is an operator built from the measurements, and u is the total field for the reference index n.

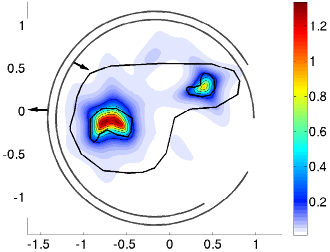
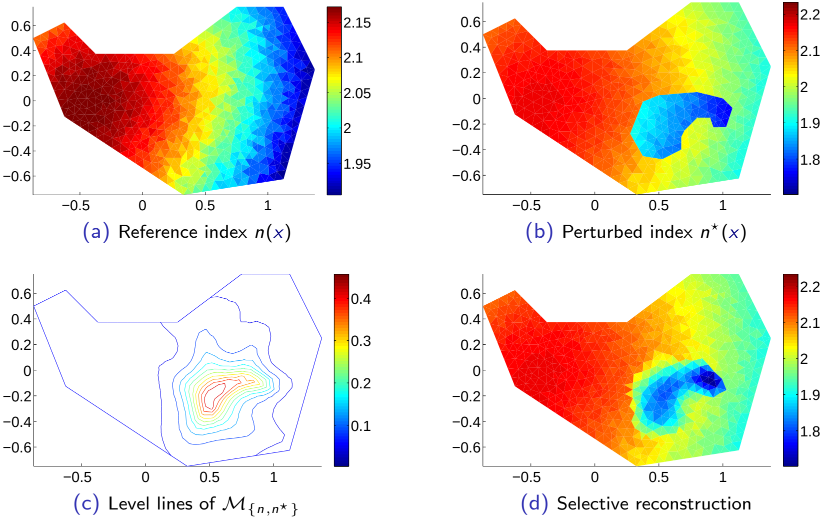


Figure: Plot of $\mathcal{M}_{\{n,n^\star\}}(x)$ for a 2D object with two defects



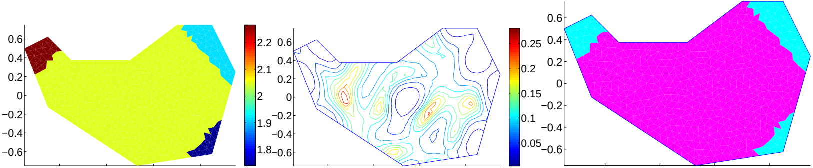
(a) Reference index $n(x)$

(b) Perturbed index $n^\star(x)$

(c) Level lines of $\mathcal{M}_{\{n,n^\star\}}$

(d) Selective reconstruction

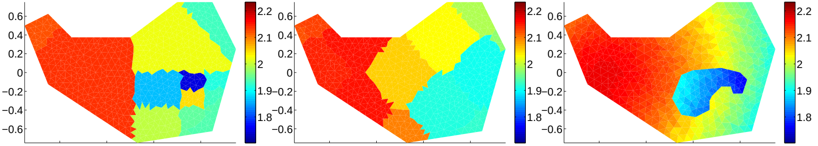
Figure: Reconstruction of a perturbed index



(a) Reconstruction $n_1(x)$ with 4 parameters

(b) Plot of $\mathcal{M}_{\{n_1, n^\star\}}(x)$

(c) Selection of a zone to divide



(d) Reconstruction $n_4(x)$ with 13 selected parameters

(e) Reconstruction with 13 parameters uniformly distributed

(f) Actual index $n^\star(x)$

Figure: Adaptive refinement

Uniqueness of the solution

Usual reconstruction of $n^\star(x)$:

$$\min J(n) := \|Simulation(n) - Observations(n^\star)\|^2_{L^2(\Gamma_m)}$$

Theorem

- $n(x), n^\star(x) \in \mathbb{R}$
- $(n - n^\star)(x) > 0$ or < 0
- Incoming and measurement directions covering the whole unit sphere

$$\boxed{\mathcal{M}_{\{n,n^\star\}}(x) = 0 \iff n(x) = n^\star(x).}$$

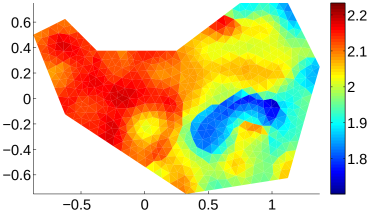


Figure: Reconstruction of $n^\star(x)$ by minimization of $J_{\mathcal{M}}(n) := \|\mathcal{M}_{\{n,n^\star\}}\|^2_{L^2(O)}$

Achievements

- Localization of defects
- Reconstruction of a perturbed index
- Adaptive refinement
- New reconstruction approach

Perspectives

- Extension of the localization to limited aperture data and absorbing media
- Motion detection in inhomogeneous media
- Free domain decomposition through the new reconstruction approach
- L^1-norm minimisation

Thank you for your attention