

# Applications of defects localization for the reconstruction of an acoustic refraction index

Y. Grisel\*

*Onera - The French Aerospace Lab.*

P.-A. Mazet†

*Onera - The French Aerospace Lab.*

V. Mouysset‡

*Onera - The French Aerospace Lab.*

J.-P. Raymond§

*IMT, Universit Paul Sabatier.*

## Abstract

We are investigating numerical methods to retrieve information about an acoustic scatterer's refraction index from far field measurements. Problems of this kind are commonly non linear and ill-posed.

We have developed a sampling method to localize differences, which we call defects, in the actual index when compared to a known reference index. So, we use this information to propose two separate strategies to reduce the number of parameters needed in the reconstruction of the actual refraction index. Moreover, we investigate the minimization of defects as a new approach for the complete index reconstruction. Our results are illustrated by numerical experiments.

## Keywords

Inverse problems, Acoustic scattering, Iterative methods.

## 1 Introduction

In inverse acoustic scattering, one tries to recover information about a scatterer from measurements. Penetrable scatterers are frequently referred to as inhomogeneous media and characterized by a refraction index [1, 3]. We are interested in the reconstruction of the refraction index from far-field measurements. This generally leads to iterative methods involving numerous heavy computations.

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\*Onera - The French Aerospace Lab, F-31055 Toulouse, France. e-mail: [yann.grisel@onera.fr](mailto:yann.grisel@onera.fr)

†Onera - The French Aerospace Lab, F-31055 Toulouse, France. e-mail: [mazet@onera.fr](mailto:mazet@onera.fr)

‡Onera - The French Aerospace Lab, F-31055 Toulouse, France. e-mail: [mouysset@onera.fr](mailto:mouysset@onera.fr)

§Université Paul Sabatier, Institut de Mathématiques de Toulouse, 31062 Toulouse Cedex, France. e-mail: [jean-pierre.raymond@math.univ-toulouse.fr](mailto:jean-pierre.raymond@math.univ-toulouse.fr)

We have extended the Factorization method [2] to localize the differences between the actual index and a known reference index. These differences will be called defects. Thus, differences between some computed index and the actual index can be seen as defects and we use this information in the reconstruction of the actual index.

## 2 Enhancement of iterative strategies for index reconstruction by selective focusing

First, we propose a strategy to reconstruct a perturbed version of some reference index. These perturbations are treated as defects and thus, can be localized. So, only the parameters corresponding to these defects need to be reconstructed. This naturally provides a substantial reduction in computational costs.

Secondly, we propose an iterative refinement strategy to compute a more precise reconstruction of a refraction index with few parameters. Each step of this strategy has two stages. First, we refine the zone containing to the most contrasting defect. Then, the reconstruction is computed with this new set of parameters. This leads to an approximation of the actual index with a constrained number of parameters positioned to fit as much as possible the geometry of this index.

## 3 Index reconstruction by defects minimisation

Lastly, another interpretation for our defects localization result, is that the absence of defects means that the actual index is equal to the reference index. Therefore, we propose a new way to reconstruct an index of refraction by looking for the reference index such that the actual index presents no more defects. We compare this approach to the classical reconstruction which consists in matching directly the simulation with the measurements. Also, we show some numerical results obtained with a Gauss-Newton method coupled with a total variation regularization [4].

## References

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