ECCM 2010
IV European Conference on Computational Mechanics
Palais des Congrès, Paris, France, May 16-21, 2010

A refraction index retrieval method using the Factorization Method for acoustics.

Y. Grisel[1], P.A. Mazet[2], V. Mouysset[3], J.P Raymond[4]

[1] *ONERA Toulouse, Université Toulouse III, France, yann.grisel@onera.fr*
[2] *ONERA Toulouse, Université Toulouse III, France, mazet@onera.fr*
[3] *ONERA Toulouse, mouysset@onera.fr*
[4] *Université Toulouse III, France, raymond@math.univ-toulouse.fr*

We are interested in numerically retrieving a scatterer's acoustic refraction index. Thus we consider the Helmholtz equation with refractive index $n(x)$ for a scattered field u^s, modeling the diffraction of an incident wave h by a scatterer $D = \text{support}(1-n(x)) \subset \mathbb{R}^d$ (d=2 or 3) :

$$\begin{cases} \Delta u^s + k^2 n(x) u^s = -k^2 n(x) h, \quad x \in \mathbb{R}^d \\ \text{Sommerfeld radiation condition on } u^s. \end{cases} \tag{1}$$

Provided some data u_0 measured on an observation surface Σ, we look for $n(x)$ minimizing the following least squares problem :

$$\mathcal{J}_0(n) := \|\mathcal{F}(n) - u_0\|^2_{L^2(\Sigma)}, \tag{2}$$

with $\mathcal{F} : n \mapsto u^s|_\Sigma$ and u^s being the solution to (1). This problem is ill-posed and non-linear. Following [1], we have investigated a coupled Gauss-Newton and Tychonov regularization method. This provides a sequence $n_{p+1} = n_p + \delta n_p$ converging locally to a solution for (2), δn_p being the minimizer of $\tilde{\mathcal{J}}_\alpha(\delta n)$:

$$\tilde{\mathcal{J}}_\alpha(\delta n) := \left\|\mathcal{F}'(n_p).\delta n - (u_0 - \mathcal{F}(n_p))\right\|^2_{L^2(\Sigma)} + \alpha \|\delta n_p\|^2_X \quad (n_p \text{ fixed}), \tag{3}$$

where $\|\cdot\|_X$ is some norm and α a regularization parameter. If $\|\cdot\|_X = \langle B\cdot,\cdot\rangle_{L^2(D)}$, then δn_p is obtained as the solution of the normal equation

$$\left(\mathcal{F}'(n_p)^\star \mathcal{F}'(n_p) + \alpha B\right) \delta n_p - \mathcal{F}'(n_p)\,(u_0 - \mathcal{F}(n_p)) = 0. \tag{4}$$

Obviously, the convergence is influenced by the choice of $\|\cdot\|_X$. Hence, this paper investigates the build of an $L^2(D)$ weighted space, whose norm will be $\|\cdot\|_X$, from a fast domain reconstruction method such as the Factorization Method [2]. It provides a characteristic function of D in terms of a constrained minimization problem :

$$\forall z \in \mathbb{R}^d,\ z \in D \iff W_n(z) := \inf\left\{\left|\langle F_n^\infty \psi, \psi\rangle_{L^2(\Gamma)}\right|,\ \langle \Phi_z^\infty, \psi\rangle_{L^2(\Gamma)} = 1\right\} > 0, \tag{5}$$

where F_n^∞ is the far field operator for (1), Γ a part of the unit sphere S_1^d and Φ_z^∞ Green's function far field for (1) with $n(x) = 1$ (free space).

Moreover, place now an inhomogeneity Ω into D, and design by $\tilde{n}$ this modified scatterer's refraction index. Then we can construct a new "characteristic" function $\tilde{W}_{\{\tilde{n},n\}}$ by using the substraction of the far fields induced by $\tilde{n}(x)$ and $n(x)$:

$$\tilde{W}_{\{\tilde{n},n\}}(z) := \inf\left\{\left|\langle (F_{\tilde{n}}^\infty - F_n^\infty)\psi, \psi\rangle_{L^2(\Gamma)}\right|,\ \langle \Phi_z^\infty, \psi\rangle_{L^2(\Gamma)} = 1\right\} > 0. \tag{6}$$

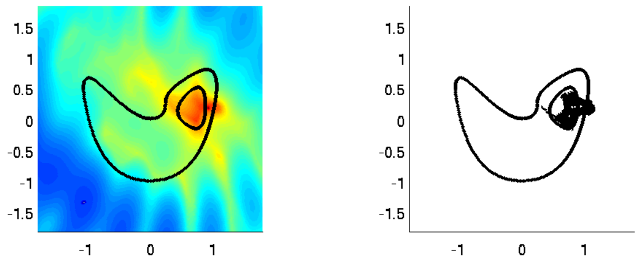

FIG. 1 – Localization of inclusion and gradient of $\tilde{W}_{\{\tilde{n},n\}}$.

This function seems to give a close representation of the shape of Ω (see [3]), as seen on Fig. 1.

The straightforward norm of the $L^2(D)$ weighted space which measure is defined by $\tilde{W}_{\{\tilde{n},n\}}$ is then taken as $\|\cdot\|_X$. Reconstruction results , comparable to those with a BV norm (see [1]), are encouraging. Fig. 2 shows an example for $n(x) = 2.5$ when $x \in B([0.3, 0.3], 0.4)$ and $n(x) = 3 + 0.1i$ elsewhere (of course $n(x) = 1$ outside of D).

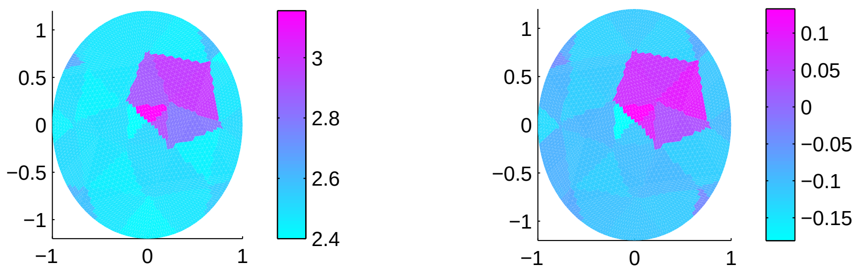

FIG. 2 – Reconstruction of $n(x)$ with the weighted $L^2(D)$ norm (real and imaginary parts).

At last, as $\tilde{W}_{\{\tilde{n},n\}}$ provides a local information on the scatterer, we investigate an adaptive refinement process for the reconstruction. This is done in order to enable more precise reconstructions without increasing the computational costs. Fig. 3 shows the areas where $n(x)$ will be looked for as a piecewise constant complex function, and the corresponding computed values ($n(x) = 1.3 + 0.4i$ when $x \in B([0.4, 0.4], 0.1)$ and $n(x) = 1.6 + 0.2i$ elsewhere).

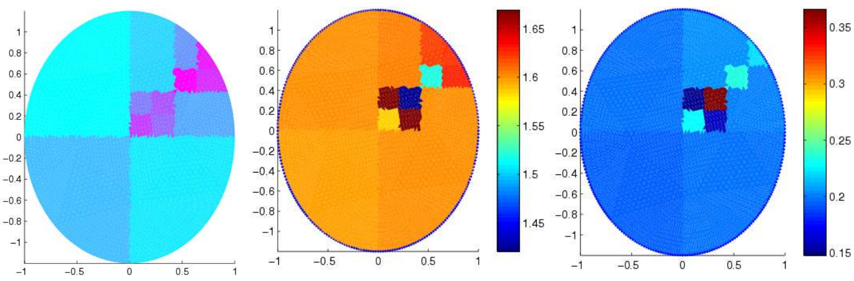

FIG. 3 – Adaptive mesh after 5 iterations (left), real (center) and imaginary (right) parts of $n(x)$

Références

[1] L. Ségui. *Sur un problème inverse en diffraction d'ondes : identificaiton de permittivités complexes d'un matériau à partir de données du champ proche.* PhD thesis, Université Toulouse III, 2000.

[2] A. Kirsh. New characterizations of solutions in inverse scattering theory. *Applicable Analysis*, 76 :319–350, 2000.

[3] Y. Grisel. An application of the factorization method to the detection of inclusions in acoustics. In *Waves 2009 proceedings*, pages 290–291. INRIA, 2009.